particles further from the wall than $\bar{y}(x)$ will have left the source later (since they are travelling quicker) than most of those particles nearer the wall than $\bar{y}(x)$, but it is not obvious why these two effects should cancel *exactly*, which is equivalent to what Shlien and Corrsin assumed. The two different graphs might have the same general shapes, but without additional argument it is not possible to justify the use of the graph of $\bar{y}(x)$ against x to determine the constants b and c appearing in equations (3) and (4).

These comments can be illustrated from the results of a theory by Townsend [6, 7] which is summarized on p. 361–364 of his book [8]. Townsend argued that, in the type of experiment considered by Shlien and Corrsin [supposing as always that conditions (i) to (iv) above hold], $\bar{\theta}(x, y)$ is approximately self-similar with the form

$$\bar{\theta}(x, y) = -\frac{Q}{\kappa_0 u_*}\left[\frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}x}\right]\eta f'(\eta) \text{ where } \eta = \frac{y}{l(x)}, \tag{6}$$

with Q being the strength of the line source, κ_0 being a constant (equal to b and κ if Reynolds analogy holds) and $f(\eta)$ is an unknown decreasing function whose integral from 0 to ∞ is 1. Townsend showed that equation (6) is consistent with the governing equations for large l/y_0 provided

$$l\left[\log\left(\frac{l}{y_0}\right) + \int_0^\infty f(\eta)\log\eta\,\mathrm{d}\eta\right] = \kappa\kappa_0 x. \tag{7}$$

Using equations (1) and (6) it follows that in Townsend's theory

$$\bar{y}(x) = l(x)\frac{\int_0^\infty \eta^2 f'(\eta)\,\mathrm{d}\eta}{\int_0^\infty \eta f'(\eta)\,\mathrm{d}\eta} = 2l(x)\int_0^\infty \eta f(\eta)\,\mathrm{d}\eta, \tag{8}$$

using the above condition on the integral of f from 0 to ∞. Townsend considered three reasonable particular forms for f which lead to the following results for $l(x)$ and $\bar{y}(x)$:

$$-\eta f' = \begin{cases}1, & \eta < 1\\ 0, & \eta > 1\end{cases}; \quad l\left[\log\left(\frac{0.37l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = \tfrac{1}{2}l(x); \tag{9a}$$

$$-\eta f' = \mathrm{e}^{-\eta}; \quad l\left[\log\left(\frac{0.56l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = l(x); \tag{9b}$$

$$-\eta f' = \begin{cases}1-\tfrac{1}{2}\eta, & \eta < 2\\ 0, & \eta > 2\end{cases}; \quad l\left[\log\left(\frac{0.45l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = \tfrac{2}{3}l(x). \tag{9c}$$

Thus the relation between $l(x)$ and x in each of the equations (9a) to (9c) is of the same form as the relation between $\bar{Y}(t)$ and $\bar{X}(t)$ in equation (4) (and indeed all four relations are approximately the same far from the source). However, and this strikingly illustrates the principal point of the previous paragraph, the ratio of $\bar{y}(x)$ to $l(x)$ varies in the three examples from $\frac{1}{2}$ to 1. Evidence quoted in [8] suggests that the true distribution is intermediate between those in equations (9b) and (9c).

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SPECIFICATION OF THE CORRECT BOUNDARY CONDITIONS

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NOMENCLATURE

k,	thermal conductivity [W/cm°C];
L,	length of the cylinder [cm];
(Q/A),	heat flux at the wall [W/cm^2];
R,	cylinder radius [cm];
r,	radial location [cm];
T,	temperature [°C];
z,	axial location [cm].

In a paper published recently, Archambault and Chevrier [1] used an implicit numerical technique based on the superposition principle to solve the two-dimensional unsteady state diffusion equation for a cylinder. This method is mathematically rigorous for a linear system and appears to have been used for the first time in connection with a numerical solution to a heat conduction problem. However, two errors were introduced in their paper; and, moreover, they did not indicate that they had checked their approximate solution against a known solution. Since we have been investigating a similar problem in connection with boiling around large horizontal cylinders [2], we report on this work as it relates to the Archambault and Chevrier paper. First, the authors failed to recognize that,

$$\text{as } r \to 0 \qquad \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\partial^2 T}{\partial r^2} \tag{1}$$

which makes the first part of the diffusion equation equal to:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial r^2} \quad \text{at} \quad r = 0. \tag{2}$$

Table 1. Comparison of different methods of solutions

	Method of solution				
	Solve 1	Solve 2	Solve 3	Solve 4	Analytical
NR = NZ = 4					
Cooling time	43	51	51	43	43
T_{diff}—radial	16.38	14.04	14.04	16.38	16.38
T_{diff}—axial	19.66	16.82	16.82	19.66	19.66
Heat balance	0.998	1.210	1.210	0.998	1.000
NR = NZ = 10					
Cooling time	43	46	46	43	43
T_{diff}—radial	16.38	15.52	15.52	16.38	16.38
T_{diff}—axial	19.66	18.62	18.62	19.66	19.66
Heat balance	1.000	1.077	1.077	1.000	1.000
NR = NZ = 15					
Cooling time	43	45	45	43	43
T_{diff}—radial	16.38	15.82	15.82	16.38	16.38
T_{diff}—axial	19.66	18.98	18.98	19.66	19.66
Heat balance	1.000	1.052	1.052	1.000	1.000

Conditions: The circular surface and the face of the cylinder were submitted to a uniform heat flux. $Q/A = 20\,\mathrm{W/cm^2}$; Diameter = 12.7 cm; Length = 15.24 cm; Thermal conductivity = 3.876 W/cm°C; Density = 8.894 g/cm^3; Thermal capacity = 0.381 W · s/g °C.

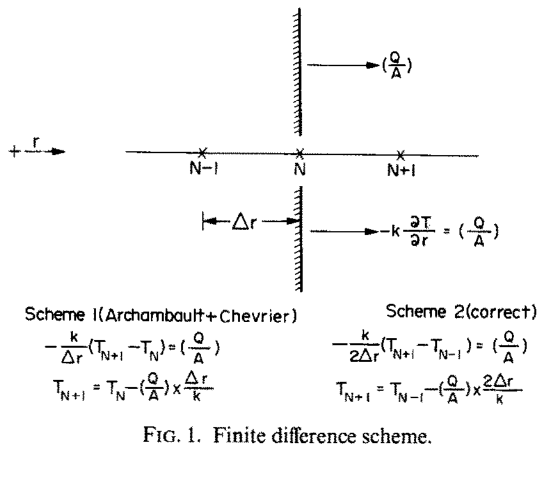

FIG. 1. Finite difference scheme.

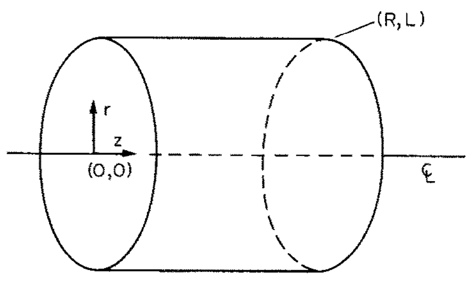

FIG. 2. Coordinate system.

The authors simply ignored the first derivative at the point of radial symmetry.

The second point relates to the approximation of the boundary condition at the cylindrical surface and at the face of the cylinder by a finite difference formulation. In deriving the finite difference equations for the mesh points at the surface, the authors have replaced the fictitious point (that is the point lying outside the surface of the cylinder) by the surface mesh point using Fourier's Conduction Law to express the heat flux at the surface (Scheme 1, Fig. 1). Although this may seem to be mathematically correct, this procedure does introduce appreciable errors. In the case of a constant heat flux situation, it is easy to derive that the gradient at the wall can only be expressed correctly in finite difference form if a central difference is used for the mesh point at the surface. If a forward difference or a backward difference are used, an appreciable error is introduced. The relative error is independent of the heat flux at the wall as well as the properties of the material; it is only dependent on the mathematical formulation used. The error does get smaller as the number of mesh points increases.

To illustrate this and also to verify and evaluate the superposition method suggested by the authors, the Alternating Direction Implicit (ADI) technique [3] was used to generate a solution to the problem. In addition, we have expressed the gradient at the wall in terms of the first interior mesh point (Scheme 2, Fig. 1) using the central difference formulation.

The problem considers a copper cylinder, initially at a uniform temperature (200°C) which is submitted to a constant heat flux (20 W/cm^2) both at the face and the circumferential surface of the cylinder. It was allowed to cool until the temperature at the centre of the cylinder became 102°C. The cooling time was long enough to allow the transients to disappear; at this time all points within the cylinder are decreasing in temperature by an equal amount with time. When the transients have disappeared, it is possible to compare the temperature distribution in the cylinder with the temperature profile as calculated from the analytical solution, viz.:

$$T(r, z) = T(0, 0) - \frac{(Q/A) \cdot R}{2k}\left(\frac{r}{R}\right)^2 + \left(\frac{Q/A \cdot L}{k}\right)\left[\frac{z}{L} - \left(\frac{z}{L}\right)^2\right]$$

It is also possible, at the end of the cooling period to perform an overall heat balance by which the total enthalpy change is compared with the total heat which should have crossed the surfaces due to the applied heat flux. The results are presented in Table 1. Table 2 gives a description of each of the numerical methods used.

It is obvious from the results obtained that the superposition method used by Archambault and Chevrier is valid as it gives the same results as a more conventional method

Table 2.

Method of solution	Numerical method	Scheme (ref. Fig. 1)
Solve 1	ADI	2
Solve 2	Superposition	1
Solve 3	ADI	1
Solve 4	Superposition	2

usually used as long as a central difference (Scheme 2) is used for the mesh point at the surface. Moreover, the superposition method offers the advantage of being simpler to program than an ADI technique as well as to take roughly 20% less execution time. In their paper, Archambault and Chevrier used 60 mesh points which would give an error in the overall heat balance and in the cooling time of about 10%.

This analysis emphasizes the importance of testing any numerical solution technique either directly by comparing its solution with an analytical solution and/or indirectly by energy or other balances.

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REJOINDER

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SUITE à votre lettre du 16 Mai 1977, nous nous permettons d'apporter less commentaires suivants:

Tout d'abord, il nous faut signaler que le but de notre travail n'était pas de développer une nouvelle méthode numérique mais plutôt de construire un outil simple et cohérent destiné à l'étude et à la détermination des caractéristiques de vaporisation d'un liquide de trempe. De plus, les remarques de Thibault et Hoffman, bien que justifiées, ne remettent pas en cause le fond de notre article ni la validité de la méthodologie proposée.

Le premier point signalé par ces auteurs est théoriquement juste mais on peut montrer que son incidence sur les résultats numériques est limitée.

Le second point concerne la représentation de la condition limite de transfert de chaleur en surface. L'utilisation d'une différence centrée est en effet mieux adaptée lorsque les gradients de température en surface ne sont pas trop importants. Dans le cas inverse, la différence centrée n'entraîne qu'une faible variation des résultats.

Nous sommes très satisfaits de la comparaison des méthodes numériques effectuées par Thibaut et Hoffman qui montre, qu'après une très légère modification, la méthode de superposition que nous proposons est aussi précise et considérablement plus efficace que la méthode classique dite implicite aux directions alternées.

Veuillez agréer, Monsieur le Professeur, l'expression de nos considérations distinguées.

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